2010 NCTS/TPE-TIMS Mini-Course and Workshop on Geometric and Complex Analysis

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ABSTRACTS

National Center for Theoretical Sciences
Taida Institute for Mathematical Sciences
Dept. of Mathematics, National Taiwan University

Geometric Analysis on the Heisenberg Group (I, II, III)

[Mini-course/Workshop]

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The Heisenberg group $\mathbf{H}_1$ is the simplest non-commutative nilpotent Lie group which has been studied extensively by many mathematicians in the past 30 years. Here $\mathbf{H}_1 = \big\{(x, y, t) \in \mathbb{R}^3\big\}$ with group law "$\circ$":

$$(x_1, y_1, t_1) \circ (x_2, y_2, t_2) = \big(x_1 + x_2, y_1 + y_2, t_1 + t_2 + 2(y_1x_2 - y_2x_1)\big).$$

The vector fields

$$X = \frac{\partial}{\partial x} + 2y\frac{\partial}{\partial t}, \qquad Y = \frac{\partial}{\partial y} - 2x\frac{\partial}{\partial t},$$

are left invariant with respect to the group law. The group $\mathbf{H}_1$ and its sub-Laplacian $\Delta = X^2 + Y^2$ are at the cross-roads of many domains of analysis and geometry: nilpotent Lie group theory, hypoelliptic second order partial differential equations, strongly pseudoconvex domains in complex analysis, probability theory of degenerate diffusion process, subRiemannian geometry, control theory and semiclassical analysis of quantum mechanics. In this series of talks, we shall give detailed discussion of geometric analysis on this group from two different point of views.

The first one is singular integral operators. We give a generalization to the Heisenberg group of the well-known characterization in $\mathbb{R}^n$ of Calderón-Zygmund singular integrals as Mihlin-Hörmander multipliers. The analogous operators we consider on the Heisenberg group are those convolution operators whose kernels are radial, singular only at the origin, and satisfy the standard Calderón-Zygmund-type regularity and cancellation conditions associated to the automorphic one-parameter dilations.

In particular this class of operators includes the Cauchy-Szegö projection, which plays a role on the Heisenberg group as distinguished as that of the Hilbert transform on the real line $\mathbb{R}$. We give an exact characterization of this class of operators as a certain class of joint spectral multipliers of the sub-Laplacian and the central derivative.

Unlike on Riemannian manifolds where one may measure the velocity and distances in all directions, on the Heisenberg group there are directions where we cannot say anything using direct methods. In this case, an important role is played by the distribution generated by the linearly independent vector fields X and Y:

$$\mathcal{H} : \mathbf{x} \to \mathcal{H}_{\mathbf{x}} = \mathrm{span}_{\mathbf{x}}\{X, Y\}, \qquad \mathbf{x} = (x, y, t) \in \mathbf{H}_1.$$

As $[X, Y] = -4\partial_t \notin \mathcal{H}$, the horizontal distribution $\mathcal{H}$ is not involutive, and hence, by Frobenius theorem, it is not integrable, *i.e.,* there is no surface locally tangent to it. A vector field V is called *horizontal* if and only if $V_{\mathbf{x}} \in \mathcal{H}_{\mathbf{x}}$, for all $\mathbf{x}$. A curve $c : [0, 1] \to \mathbb{R}^3$ is called horizontal if the velocity vector $\dot{c}(s)$ is a horizontal vector fields along $c(s)$. Horizontality is a constraint on the velocities and hence, it is also called in the literature nonholonomic constraint.

In the second part of out talks, we shall construct many horizontal objects, *i.e.,* a geometric objects which can be constructed directly from the horizontal distribution and the subRiemannian metric defined on it. The main goal is to recover the external structure of the space, such as the missing direction ∂_t by means of horizontal objects. Using these information, we may construct the heat kernel and hence the fundamental solution for the sub-Laplacian.

Liouville-type estimate and weak sub-laplacian comparison theorem in a complete pseudohermitian 3-manifold

[**Workshop**]

Shu-Cheng Chang

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In this talk, we show that the natural CR analogue of Liouville–type theorem holds for the positive pseudoharmonic function in a complete pseudoher–mitian 3-manifold with vanishing torsion and nonnegative Tanaka–Webster scalar curvature. In particular, we recapture Koranyi and Stanton's result for Liouville–type theorem where their method is only worked in Heisenberg groups. The key is to obtain the weak sub-Laplacian comparison theorem and CR volume growth estimates. This is part of joint work with J. Cao, J.-Z. Tie and C.-T. Wu.

Complete Noncompact Manifolds with Positive Spectrum

[Mini-course]

Jui-Tang Chen

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In this talk, we consider p-capacity, p-harmonic function, p-hyperbolic and p-parabolic on complete noncompact Riemannian manifolds, here we refer to the papers of A. Grigorýan, M. Troyanov, I. Holopainen, L. Ni, etc. One of the interesting questions is to consider p-harmonic function on complete compact manifold with positive spectrum. In 2001, P. Li and J. Wang proved the decay estimate of bounded harmonic function on an end, and they use this property to show the decay estimate of volume on end and the splitting type theorem whenever the Ricci curvature is bounded from below by the lower bound of the spectrum. According to their results, we will show our recent work on manifolds with positive spectrum.

An ADM-like mass in Cauchy-Riemann geometry

[**Workshop**]

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We define an ADM–like mass, called p-mass, for an asymptotically flat pseudohermitian manifold. The p-mass for the blow-up of a closed pseudohermitian manifold is identified with the first nontrivial coefficient in the expansion of the Green function for the CR Laplacian. We deduce an integral formula for the p-mass. The p-mass is nonnegative if the Webster-Tanaka curvature is nonnegative and the Paneitz-like operator is nonnegative in dimension 3. We show the existence of nonembeddable CR 3-manifolds having nonpositive Paneitz-like operator through a second variation formula. We also discuss the zero p-mass situation and the relation to the CR Yamabe problem.

Convexity package for moment maps on contact manifolds

[Workshop]

River Chiang

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Let a torus T act effectively on a compact connected cooriented contact manifold, and consider the natural moment map on the symplectization. We prove that, if $dim T > 2$, the union of the origin and the moment image is a convex polyhedral cone, the nonzero level sets of the moment map are connected (while the zero level set can be disconnected), and the moment map is open as a map to its image. This is a joint work with Yael Karshon.

Integral geometry, theorems of Morera-type, and cross-sectional comparison problems

[Workshop]

Eric Grinberg

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We consider problems i) of determining an unknown function, ii) of detecting a solution o a differential equation and iii) comparing geometric properties, e.g., areas or volumes, of two objects, by means of integrals over lower dimensional varieties. The seemingly different nature of these problems notwithstanding, integral geometry provides a common framwork for their consideration.

Topics in 3-dimensional contact topology

[Mini-course]

Otto van Koert

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In this mini-course we shall give an introduction to contact topology with emphasis on 3-dimensional contact manifolds. We will introduce the important dichotomy between tight and overtwisted contact manifolds, and discuss some applications of J-holomorphic curves, such as non-fillability of overtwisted contact manifolds. We shall also give a survey of convex surface theory and its application to the classification of contact manifolds in some simple cases.

The geometry of J-holomorphic mappings and contact metric manifolds

[Mini-course]

Takanari Saotome

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In this talk, we will study the geometry of strongly pseudo-convex manifolds and J-holomorphic mappings between strongly pseudo-convex manifolds.

I will do my talk with four parts. In the first two parts of my talk, we will review some basic topics in contact geometry and CR geometry, as an introduction.

I will also discuss some known results which is related to CR geometry and the geometry of J-holomorphic mappings.

In the talk of last two hours, I will introduce the parts of my present research of J-holomorphic mappings between strongly pseudo-convex manifolds. In particular, I investigate the removable singularity theorem and the sub-ellipticity of J-holomorphic mappings.

The Cauchy-Riemann equations in complex analysis and geometry

[Mini-course]

Mei-Chi Shaw

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The Cauchy-Riemann equations play a central role in the function theory of both one and several complex variables. In this mini-course, we will discuss some old and new results on the existence and regularity of solutions of the Cauchy-Riemann equations, the tangential Cauchy-Riemann equations and their applications to function theories of in complex analysis and geometry.

We will first discuss some old and new results on solutions of the $\overline{\partial}$-equation in complex Euclidean spaces and on complex manifolds using both the Hilbert space approach and the integral kernel method. The Hilbert space approach uses harmonic integrals from potential theory of domains with boundary, first developed by Kohn and Hörmander in the 60's. The integral kernel approach developed by Grauert-Lieb and Henkin in the 70's generalizes the classical Cauchy integral formula in one complex variable to several variables. We will also discuss recent developments on complex manifolds with emphasis on the role that curvature plays in the solutions for the $\overline{\partial}$ operator.

The second topic will be on the existence and regularity of the tangential Cauchy-Riemann equation on CR manifolds. Both local and global existence results will be examined. We will prove the closed-range property of the $\overline{\partial}_b$–operator. In particular, we will present new Hölder and L^p estimates for $\Box_b$ and discuss their applications. The topics will be divided into four lectures:

1. The Levi problem and the Cauchy-Riemann equations in $\mathbb{C}^n$. Introduction to the classical L^2 results on pseudoconvex domains which leads to the solution to the Levi problem. The $\overline{\partial}$-Neumann problem with or without weights. Hörmander's

L^2 existence theorem. Boundary regularity for $\overline{\partial}$.

2. The Cauchy-Riemann equations in curved manifolds. Cauchy-Riemann equations on domains in complex manifolds which are not Stein. In particular, we will discuss the $\overline{\partial}$-equation on domains in complex projective spaces (positively curved manifolds) and product domains. This extends the classical results of the Serre duality, Bochner-Kodaira and the Kunneth formula to open manifolds with boundary.

3. The tangential Cauchy-Riemann complex. Subelliptic estimates for the tangential Cauchy-Riemann complex on strongly pseudoconvex boundary, L^2 theory for the tangential Cauchy-Riemann complex on pseudoconvex boundary and integral formula for $\overline{\partial}$ and $\overline{\partial}_b$ on convex domains.

4. Embedding CR manifolds and CR geometry. Embedding compact strongly pseudoconvex CR manifolds with a real dimension of at least 5. Local embedding problems and the closed range property for $\overline{\partial}_b$. Function theory on positively or negatively curved manifolds. We will end the lectures by presenting a list of open problems.

The Cauchy-Riemann equations on complex manifolds

[Workshop]

Mei-Chi Shaw

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We will first discuss some old and new results on solutions of the $\overline{\partial}$-equation In this talk we study the closed range property and boundary regularity of the Cauchy-Riemann equations on domains in complex manifolds with emphasis on the role of the curvature condition. We will discuss the case for pseudoconcave domains and the null space of the $\overline{\partial}$-Neumann operator. Recent results for the Cauchy-Riemann equations on product domains and an L^2 version of the Serre duality on domains in complex manifolds will be discussed (joint work with Debraj Chakrabarti).

Mean curvature flow and the defomation of map between manifolds

[**Mini-course**]

Mao-Pei Tsui

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In this mini-course we shall give an introduction to higher codimensional mean curvature flow (MCF) with emphasis on the cases of graphic MCF. We will discuss how MCF can be used to study several geometric problems related to maps between manifolds.

F-Yang-Mills Fields, F-Harmonic Geometry, and Sharp Geometric Inequalities on Manifolds

[Workshop]

Shihshu Walter Wei

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Let $F : [0,\infty) \mapsto [0,\infty)$ be a strictly increasing C^2 function with $F(0) = 0$. We unify the concepts of F-harmonic maps, Yang-Mills Fields, minimal surfaces, and maximal spacelike hypersurfaces, and introduce F-Yang-Mills fields, and generalized Yang-Mills-Born-Infeld fields (with the plus sign or with the minus sign) on manifolds. When $F(t) = t, \frac{1}{p}(2t)^{\frac{p}{2}}, \sqrt{1+2t}-1$, and $1-\sqrt{1-2t}$; the F-Yang-Mills field becomes an ordinary Yang-Mills field, p-Yang-Mills field, a generalized Yang-Mills-Born-Infeld field with the plus sign, and a generalized Yang-Mills-Born-Infeld field with the minus sign on a manifold respectively. We will also discuss F-harmonic geometry and sharp geometric inequalities on manifolds. Some applications to geometry, topology, differential equations, and geometric flows will be considered.

The K$\breve{a}$hler-Ricci flow

[Mini-course]

Ben Weinkove

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We will discuss and survey results on the K$\breve{a}$hler-Ricci flow on a compact K$\breve{a}$hler manifold. We will cover several topics in the course, due to a number of authors, including: the K$\breve{a}$hler-Ricci flow for arbitrary K$\breve{a}$hler classes, minimal surfaces of general type, convergence properties of the flow and open problems.

Contracting exceptional divisors by the K$\breve{a}$hler-Ricci flow

[Workshop]

Ben Weinkove

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We give a criterion under which a solution of the K$\breve{a}$hler-Ricci flow contracts exceptional divisors on a compact manifold and can be uniquely continued on a new manifold. This is a joint work with Jian Song.

Geometric Problems in CR Manifolds

[Workshop]

Chin-Tung Wu

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The titles of the mini-course:

(1) The CR Bochner formula and Obata Theorem (1 hr)
Abstract: We derive the CR Bochner formula and discuss the lower bound for the first nonzero eigenvalue of the sublaplacian. We show that under some conditions on covariant derivatives of pseudohermitian torsion, the lower bound is achieved.

(2) The Li–Yau Harnack Inequality for the CR Heat Flow (1 hr)
Abstract: By using the CR Bochner formula and the arguments of Li–Yau, we derive the Harnack inequality for the positive solution of the CR heat equation on a closed pseudohermitian $(2n+1)$–manifold.

(3) The CR Yamabe Flow (1hr)
Abstract: We deform the contact form by the amount of the Tanaka-Webster curvature minus its mean value on a closed spherical CR 3–manifold. We show that if a contact form evolves with free torsion from initial data with positive Tanaka-Webster curvature, then the normalized Yamabe flow exists for all time and converges smoothly to, up to the CR automorphism, a unique limit contact form of constant Webster scalar curvature as $t \to \infty$.

(4) The Q–curvature Flow on a closed Pseudohermitian 3–manifold (1 hr)
Abstract: Let M be a closed pseudohermitian 3–manifold. We study the normalized Q–curvature flow. We show that the solution exists for all time and converges smoothly to a contact form of zero Q–curvature.