Jing Yu

Position
Chiar Professor of Department of Mathematics, NTU

Research Fields
Number Theory, Arithmetic Geometry

Research Interests

In the direction of arithmetic geometry over function fields of a fixed positive characteristic p, one can always probe deeper than in the case of number fields. Here the Riemann Hypothesis was proved by Weil-Deligne long time ago; the Langlands conjecture was proved by Drinfeld-Lafforgue 10 years ago. With such a firm foundation, we can ask various questions in arithmetic which are completely out of reach in the classical world of characteristic zero.

I have been interested in determine all the algebraic relations among transcen- dental invariants (the various *periods*) which arise naturally in arithmetic. In pos- itive characteristic, all these invariants depend on the base finite field $\mathbb{F}_q$. We are interested in comparing these invariants when the base fields vary in the same characteristic p. We also want to determine the algebraic relations among periods and quasi-periods of non-isogenous Drinfeld modules (defined over $\bar{k}$, the algebraic closure of the rational function field k in one variable over $\mathbb{F}_q$). Motivic transcendence theory as described in Grothendieck's program is realized in positive characteristic world by the method of t-motives (Anderson-Papanikolas), which is a reformulation of my t-module method 10 years ago. This method opens a door for proving algebraic independence which does not exist in characteristic zero. This is a multi- variable Galois theory with algebraic Galois groups. We are able to go from linear independence to algebraic independence precisely because we may use the structure of these algebraic Galois groups as a tool, besides merely analysis. In this positive characteristic motivic theory, another important advantage is that the algebraic Galois groups admit explicit descriptions via Frobenius difference equations: the algebraic Galois group is actually isomorphic to the difference Galois group, as an analogue of the Picard-Vessiot theory in differential field extensions.

In another direction, I am also interested in arithmetic of families of curves over function fields, including Drinfeld modular curves, elliptic curves over function fields, and various Shimura curves. We apply automorphic forms of Drinfeld types to arithmetic questions concerning these curves, and in particular to the questions that relate special values of L-functions with special rational points on the curves. The game starts from fixing a place of the function field and call it the infinity. Automorphic forms which we are interested in has to be special with respect to this place, in the sense that the associated GL_2-representation at ∞ is the so- called special representation. For arithmetic of function fields, this is a natural step although this infinite place can certainly be switched.

Finally we are also particularly interested in phenomena of function field arithmetic which are completely non-classical, in the sense that they do not have close analogues in arithmetic geometry over number fields.

Research interests of other members of our group: Jeng-Daw Yu, interested in cohomology, in particular rigid cohomology in positive characteristic. He studies application of various kind of cohomologies to specific families of varieties over finite fields. Ming-Lun Hsieh, interested in Iwasawa theory and the main conjecture of Iwasawa. He uses theory of automorphic representations over number fields as tool, besides the p-adic congruences. Chia-Fu Yu, an expert on arithmetic of moduli spaces of abelian varieties, and geometry Shimura varieties. He is also interested in the interaction of theory of automorphic representations with arithmetic geometry, e.g. using trace formua as a tool. He is well-versed both in the geometry side and the algebraic side of the arithmetic objects he is interested in. Chieh-Yu Chang, an expert in motivic transcendence theory in positive characteristic. He is also interested in algebraic independence of values of modular forms at algebraic points. Fu-Tsun Wei, he works on arithmetic of curves over function fields, mass formulas and class number formulas for division algebras over global fields.

Research Group

- *Jing Yu (NTU), Chair Professor*
- *Jeng-Daw Yu (NTU), Assistant Professor*
- *Ming-Lun Hsieh (NTU), Assistant Professor*
- *Chia-Fu Yu (Academia Sinica), Professor*
- *Chieh-Yu Chang (NTHU), Assistant Professor*
- *Fu-Tsun Wei (NTHU), Post-Doctor*